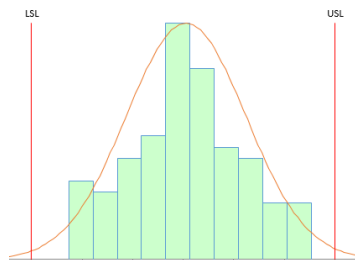


An Interactive Look at Process Capability

This month's publication takes a look at process capability – a topic we first covered almost 10 years ago. Yes, we have been producing this monthly publications for a long time!

Cpk

We have developed an interactive Excel workbook that lets you visually see how changing the average and standard deviation of your process impacts your process capability. You will be able to see visually how the process shifts versus your specifications. In addition, the workbook shows how Cp, Cpk, the sigma level, and the ppm out of specification change. You can download the workbook for your own use.



Process capability tells you how well your process meets your customer specifications. If you read much about process capability, you will have discovered that there are widely-diverging opinions on how useful this statistic is. Some people say it is not very useful – that many processes aren't normally distributed. A common complaint is that many people do not check to see if their process is in statistical control before calculating the process capability.

Many folks just calculate a process capability value each month or for each lot – not realizing that a change in the value of the process capability may be simply due to common causes of variation. In the end though, it is your customer who decides how important process capability is to you. For many customers, this is their only way of “judging” how well your process meets their specifications – even with all the caveats that go along with calculating the process capability. Like it or not, process capability calculations are here to stay.

So, this month, we provide you with a visual method of seeing how process capability changes as the average, standard deviation and specifications change. You can use the interactive workbook to help you see these changes or to help explain the impact of these changes to others. Hope you enjoy the interactive workbook.

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A Review of Process Capability: Cp and Cpk

We will briefly review two process capability indices here: Cp and Cpk. For more information on these (and on Pp and Ppk), please see our three--part series on [process capability](#) in our SPC Knowledge Base. Process capability answers the question of how well our process meets our customer's specifications. We will use LSL and USL for the lower and upper specification limits, respectively.

Remember, the process capability indices calculations require that your process be in statistical control and that the individual measurements are normally distributed.

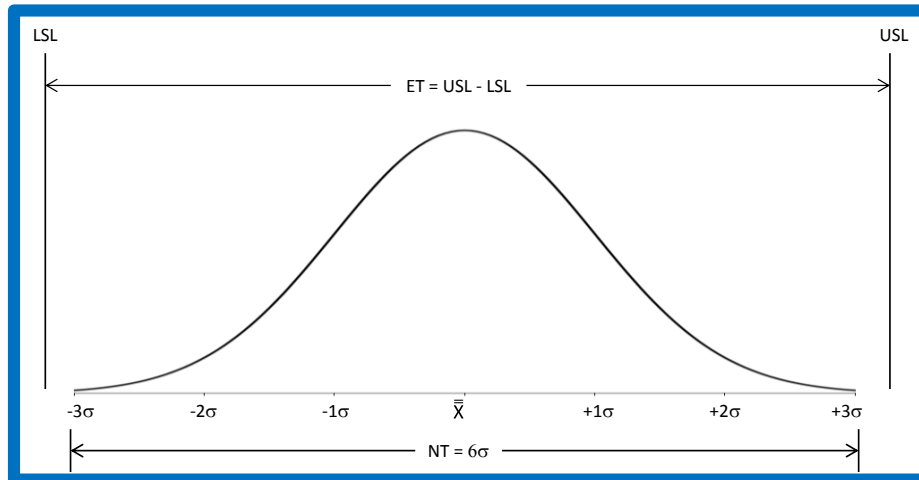
The first capability ratio, C_p , is defined as the ratio of the engineering tolerance ($USL - LSL$) to the natural tolerance (-3σ to 3σ):

C_p

$$C_p = \frac{ET}{NT} = \frac{USL - LSL}{6\sigma}$$

where σ is the process standard deviation estimated from a range control chart. Note that σ is not the calculated standard deviation. The visual presentation of C_p is shown in Figure 1.

Figure 1: C_p Index



For C_p :

- If the engineering tolerance is less than the natural tolerance (i.e., $C_p < 1.0$), the process is not capable of meeting specifications.
- If the engineering tolerance is equal to or larger than the natural tolerance ($C_p > 1.0$), the process is capable of meeting specifications.

However, a process can be capable of meeting specifications but not be meeting specifications if the process is not centered relative to the specifications. In fact, you can have a value of $C_p > 1.0$ and everything be out of specifications if the process operates completely outside the specification limits.

C_p
or
 C_{pk} ?

C_p values are not the best indicators of process capability. The value of C_p does not take into account where the process is centered. Just knowing that a process is capable ($C_p > 1.0$) does not ensure that all the product or service being received is within the specifications. A process can have a $C_p > 1.0$ and produce no product or service within specifications. In addition, C_p values can't be calculated for one-sided specifications. A better measure of process capability is C_{pk} .

C_{pk} takes into account where the process is centered. The value of C_{pk} is the minimum of two process capability indices. One process capability is C_{pu} , which is the process capability based on the upper specification limit. The other is C_{pl} , which is the process capability based on the lower specification limit. Algebraically, C_{pk} is defined as:

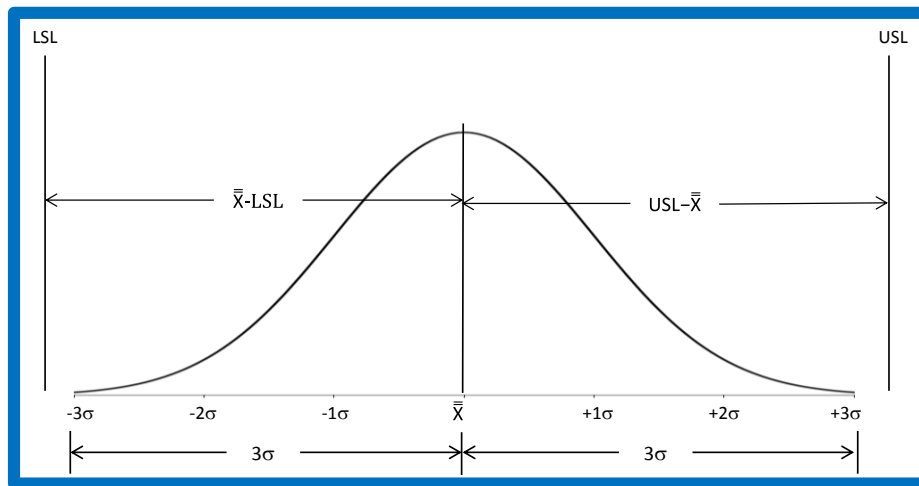
$$C_{pk} = \text{Minimum } (C_{pu}, C_{pl})$$

$$C_{pu} = \frac{USL - \bar{X}}{3\sigma}$$

$$C_{pl} = \frac{\bar{X} - LSL}{3\sigma}$$

Figure 2 shows how the Cpk values are calculated. For the area below the average, it can be seen that the Cpl is simply the ratio of $\bar{X} - LSL$ to 3σ . If that ratio is greater than one, the LSL is more than 3σ from the average. Likewise, Cpu is simply the ratio of $USL - \bar{X}$ to 3σ . If that ratio is greater than one, the USL is more than 3σ from the average. Cpk is the minimum of Cpu and Cpl. So, if Cpk is greater than 1, then no product is being produced out of specification on the high or low side. Yes, there is a small percentage of the normal curve outside $\pm 3\sigma$. This is why more and more customers are demanding higher Cpk values, e.g., 1.33.

Figure 2: Cpk Index



If there is only one specification, the value of Cpk is either Cpu or Cpl, whichever is appropriate for the specification. So, if there is only an USL, then $Cpk = Cpu$. Likewise, if there is only an LSL, the $Cpk = Cpl$.

What Happens as the Process Standard Deviation Changes?

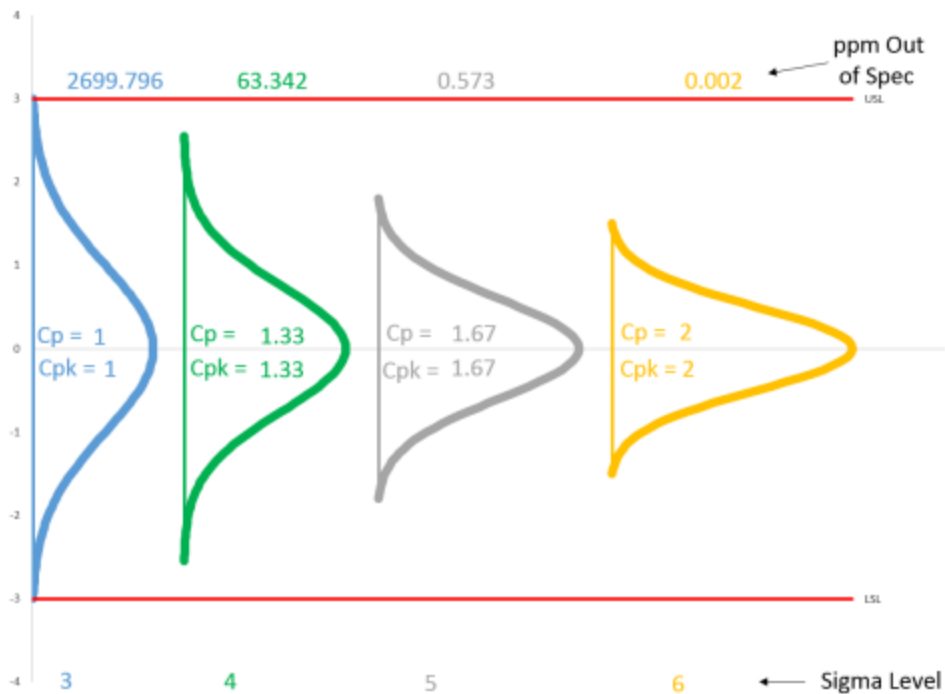


This section looks at the impact of decreasing the variation (the standard deviation) of your process on Cp, Cpk, sigma level and ppm out of specification. The results are from the interactive workbook available for download at the end of this publication. In the workbook, you can enter an average, standard deviation and specification limits. For simplicity, we will start with the following:

- Average = 0
- Standard deviation = 1
- Lower specification limit = -3
- Upper specification limit = 3

When you enter these into the workbook, the chart below is updated to reflect the values entered

Figure 3: Impact of Decreasing Variation for Centered Process



There are four normal curves on the chart. The average and specification limits are the same for each of the curves. What changes is the standard deviation. The first normal curve (blue) has the standard deviation entered, in this case 1. In the second normal curve (green), the standard deviation has been decreased by a factor of $\frac{3}{4}$. In the third normal curve (gray), the standard deviation has been decreased by a factor of $\frac{3}{5}$. In the fourth normal curve, the standard deviation has been decreased by a factor of $\frac{1}{2}$.

Four values are also given for each curve: Cp, Cpk, sigma level and ppm out of spec. Cp and Cpk are calculated using the formula above. Sigma level is simply equal to $3Cpk$. The ppm out of specification is calculated using the NORMSDIST function in Excel. You can see the formulas used by examining the worksheets in the interactive workbook.



What does Figure 3 show us? First, you can see that $Cp = Cpk$ for each of the curves. This is because the process is “centered” – the average lies halfway between the specifications. Whenever the process is centered, Cp will equal Cpk. The first curve has $Cp = Cpk = 1$. This means that the specification limits are three standard deviations away from the average. The “sigma level” is 3 for this curve. Here, we will ignore the Six Sigma practice of allowing a 1.5 sigma shift in looking at sigma level. The total out of specification for this curve is about 2700 ppm.

The standard deviation in the second curve has decreased from 1 to 0.75. What has happened to the curve? It is narrower since the standard deviation is smaller. Note that $Cp = Cpk = 1.33$. The sigma level

is now 4 – the specification limits are now four standard deviations away from the average. The out of specification has decreased from 2700 to about 63 ppm.

The standard deviation in the third curve has decreased further – to 0.6. The curve is narrower again. $C_p = C_{pk} = 1.67$. The sigma level is now 5 – the specifications are five standard deviations away from the average. The out of specification has decreased to 0.57 ppm.

The fourth curve represents that magical 6 sigma level. The standard deviation is now 0.5 and $C_p = C_{pk} = 2$. The sigma level is 6 – the specifications are six standard deviations away from the average and the out of specification is now 0.002 ppm.

So by decreasing the standard deviation by a factor of 2 for this example, the ppm out of specification dropped from 2700 to 0.002.

In addition, the interactive workbook contains a table with the detailed results. The table for this example is shown below.

Table 1: Results for Average = 0, USL = 3, LSL = -3

Sigma	1	0.75	0.6	0.5
Cp	1	1.33	1.67	2
Cpu	1	1.33	1.67	2
Cpl	1	1.33	1.67	2
Cpk	1	1.33	1.67	2
Out of Spec LSL (ppm)	1349.898	31.67124	0.286652	0.000987
Out of Spec USL (ppm)	1349.898	31.67124	0.286652	0.000987
Total Out of Spec (ppm)	2699.796	63.342	0.573	0.002
Sigma Level	3	4	5	6

From here, you can change the average, standard deviation, and specification limits to see what the impact is on the process capability results. For example, suppose you want to see what would happen if your process average has a huge shift – from 0 to 1.5. What happens to the process capability results? The results for this process shift are shown in Figure 4. The figure shows what happens for each of the four curves with this shift in the average. Note that C_p does not change. This is because the standard deviation and specifications did not change – and that is all C_p is based on.



C_{pk} and the sigma level have decreased while the ppm out of specification has increased significantly. Note that the sigma level for the fourth curve is 3. This is the curve that had the 6 sigma level before the shift. So, even with a large shift in the average, the 6 sigma level process still is producing most of the product within specifications. The table of results for the data in Figure 4 is given in Table 2.

We will not show what happens when you change specifications here. You can experiment with that in the interactive workbook.

Figure 4: Process Shift from 0 to 1.5

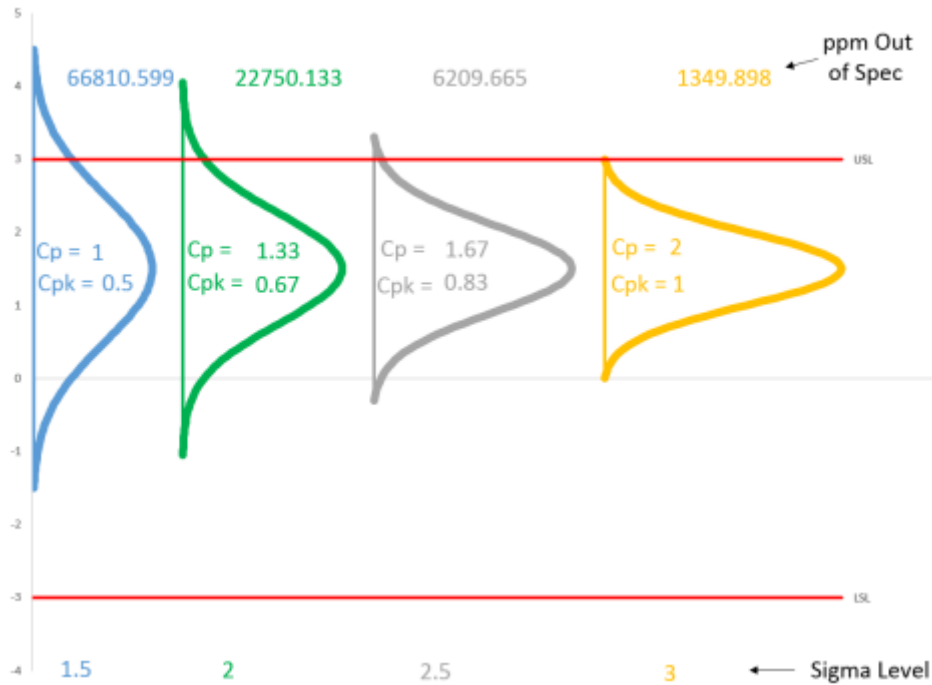


Table 2: Results from 1.5 Shift in Average

Sigma	1	0.75	0.6	0.5
Cp	1	1.33	1.67	2
Cpu	0.5	0.67	0.83	1
Cpl	1.5	2	2.5	3
Cpk	0.5	0.67	0.83	1
Out of Spec LSL (ppm)	3.397673	0.000987	3.19E-08	0
Out of Spec USL (ppm)	66807.2	22750.13	6209.665	1349.898
Total Out of Spec (ppm)	66810.6	22750.13	6209.665	1349.898
Sigma Level	1.5	2	2.5	3

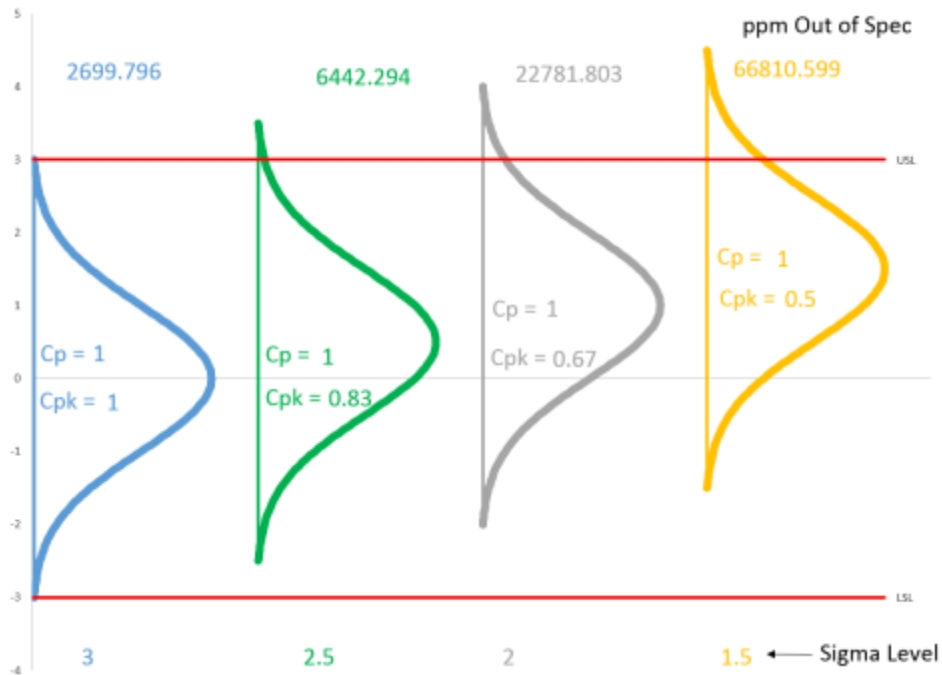
What Happens as the Process Average Shifts?



The interactive workbook also shows the impact of a changing average on the process capability when the standard deviation stays the same. The workbook includes shifts in the average of 0.5, 1.0, and 1.5 standard deviations.

Figure 5 shows the results with the same starting point as Figure 3 (average = 0, standard deviation = 1, USL = 3, LSL = -3). Note that the first normal curve is the same on both charts.

Figure 5: Shifting Average, Same Standard Deviation



As the average moves closer to the specification limit, Cpk and sigma level decrease as the ppm out of specification increases. The tabular results are shown below.

Table 3: Results for Shifting Average, Same Standard Deviation

Average	0	0.5	1	1.5
Cp	1	1	1	1
Cpu	1	0.83	0.67	0.5
Cpl	1	1.167	1.33	1.5
Cpk	1	0.83	0.67	0.5
Out of Spec LSL (ppm)	1349.898	232.6291	31.67124	3.397673
Out of Spec USL (ppm)	1349.898	6209.665	22750.13	66807.2
Total Out of Spec (ppm)	2699.796	6442.294	22781.8	66810.6
Sigma Level	3	2.5	2	1.5

We will leave it up to you to explore other situations using the interactive workbook.

Summary

This publication presented a brief review of process capability and then introduced an interactive Excel workbook that allows you to see the impact of changing the process average, process standard deviation, or specification limits on process capability. The interactive workbook looks at two different situations:

- Constant average, changing standard deviation
- Constant standard deviation, changing average

The interactive workbook contains a visual picture of how the process capability changes as well as tabular results.

You can download the [interactive workbook here](#). We hope you find it useful. Please feel free to give us suggestions on how to improve it.

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Thanks so much for reading our publication. We hope you find it informative and useful. Happy charting and may the data always support your position.

Sincerely,

Dr. Bill McNeese
BPI Consulting, LLC